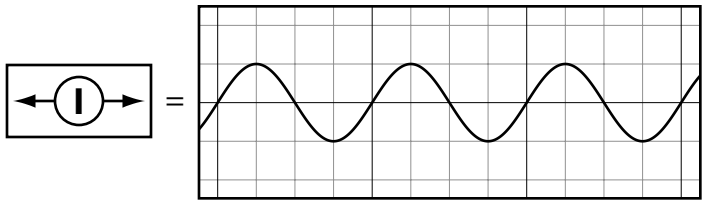
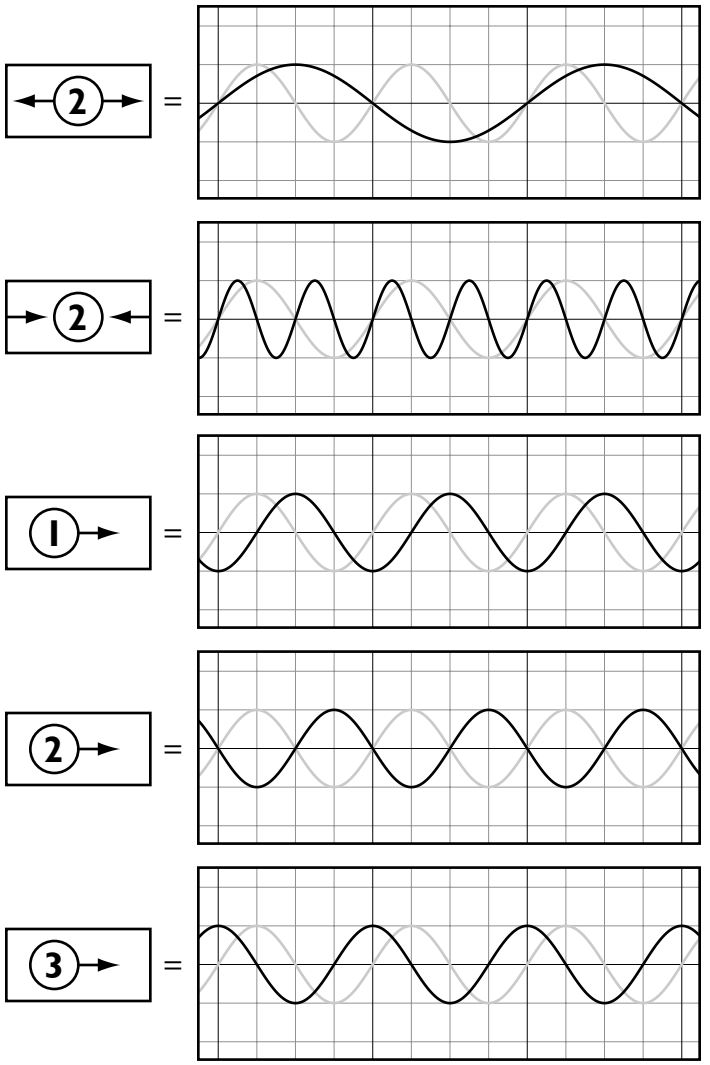
When most people think of waves, such as ocean waves, they imagine simple, regular wave motion. But waves can take on much more complicated shapes and patterns. First, simple waves can be manipulated by stretching, compressing, and shifting. Multiple simple waves can then be added together to form new waves that are complicated but still regular and repeating. These complicated waves are called "Fourier series", named after Joseph Fourier (1768-1830) who studied heat conduction.

In the following puzzles, a simple wave will be manipulated using stretchers, compressors, and shifters. Given the choices of wave manipulators, you need to determine which combination will produce the wave given.

We start with a regular wave:



Here are some examples of wave manipulators:



Using combinations of these wave manipulators, we can produce complicated waves. In this example, a simple wave that is compressed by 2 and then shifted by 3 is combined with another simple wave that is compressed by 3 and then shifted by 2 to form the complicated wave shown.